

Зачет 23

Интегрирование рациональных функций

N 49 а)

$$\int \frac{1+2t}{1-3t} dt$$

N 49 б)

$$\int \frac{t^5}{1-t} dt = \int -\frac{t^5}{t-1}$$

$$\frac{t^5-1}{t-1} = t^4 + t^3 + t^2 + t + 1$$

$$= \int (t^4 + t^3 + t^2 + t + 1 + \frac{1}{t-1}) dt = \frac{t^5}{5} + \frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} + t + \ln|t-1| + C$$

$$= \left(\frac{t^5}{5} + \frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} + t + \ln|t-1| \right) + C$$

N 49 в)

$$\int \frac{t}{(1-t)^{100}} dt = \int \frac{x}{x^{100}} dx = \int \frac{1}{x^{99}} dx$$

$$= \int \left(\frac{1}{x^{99}} + \frac{1}{x^{100}} \right) dx = \frac{x^{-98}}{-98} + \frac{x^{-99}}{-99} + C$$

$$= -\frac{(t-1)^{-98}}{98} - \frac{(t-1)^{-99}}{99} + C$$

N 49 г)

$$\int \frac{4t^3}{2t^8 - 3t^4 + 13} dt = \int \frac{4t^3 dx}{2x^2 - 3x + 13} = \int \frac{dx}{2x^2 - 3x + 13}$$

$$= \int \frac{dx}{2x^2 - 3x + 13} = \int \frac{dx}{2(x - \frac{3}{4})^2 + \frac{95}{8}}$$

$$= \frac{1}{2} \int \frac{dx}{(x - \frac{3}{4})^2 + \frac{95}{16}} = \frac{1}{2} \cdot \frac{4}{\sqrt{95}} \arctg \frac{(x - \frac{3}{4})^4}{\sqrt{95}} + C$$

$$= \frac{2}{\sqrt{95}} \arctg \frac{4x-3}{\sqrt{95}} + C$$

Метод разложения на простые дроби. (МНК)

N 50 а)

Зачетного метода МНК

$$\int \frac{t^2-1}{(t-2)(t+3)} dt = \int \frac{t^2-1}{t^2+t-6} = \int \frac{t^2-1}{t^2+t-6} \cdot \frac{t^2+t-6}{t^2+t-6} dt$$

$$= \int \left(1 - \frac{t+5}{t^2+t-6} \right) dt = t - \int \frac{t+5}{(t-2)(t+3)} dt = t - \int \frac{t+5}{(t-2)(t+3)} dt$$

$$\frac{t+5}{(t-2)(t+3)} = \frac{A}{t-2} + \frac{B}{t+3} = \frac{A(t+3) + B(t-2)}{(t-2)(t+3)} = \frac{(A+B)t + 3A-2B}{(t-2)(t+3)}$$

$$\begin{cases} A+B=1 \\ 3A-2B=0 \end{cases} \Rightarrow 5A=2 \Rightarrow A=\frac{2}{5} \Rightarrow B=\frac{3}{5}$$

$$\Rightarrow y_1 = \int \left(\frac{2/5}{t-2} + \frac{3/5}{t+3} \right) dt = \frac{2}{5} \ln|t-2| + \frac{3}{5} \ln|t+3| + C$$

Big nobis: $t - \frac{2}{5} \ln|t-2| - \frac{3}{5} \ln|t+3| + C$

N50 m $\int \frac{t^2-3}{t^2(t^2-1)} dt = \int \frac{t^2-3}{t^2(t-1)(t+1)} dt \quad \text{⊖}$

$$\frac{t^2-3}{t^2(t-1)(t+1)} = \frac{A}{t^2} + \frac{B}{t} + \frac{C}{t-1} + \frac{D}{t+1} =$$

$$= A(t-1)(t+1) + Bt(t-1)(t+1) + Ct^2(t+1) + Dt^2(t-1) =$$

$$= t^3(B+C+D) + t^2(A+C-D) + t(-B) - A$$

$$\Rightarrow \begin{cases} t^0: -A = -3 \\ t^1: -B = 0 \\ t^2: A+C-D = 1 \\ t^3: B+C+D = 0 \end{cases} \Rightarrow \begin{cases} A = 3 \\ B = 0 \\ D = 1 \\ C = -1 \end{cases}$$

$$\text{⊖} \int \left(\frac{3}{t^2} - \frac{1}{t-1} + \frac{1}{t+1} \right) dt = -\frac{3}{t} - \ln|t-1| + \ln|t+1| + C$$

N50 k $\int \frac{t^2-1}{(t-2)(t^2+t+1)} dt \quad \text{⊖} / \Delta < 0 \Rightarrow$

$$\frac{t^2-1}{(t-2)(t^2+t+1)} = \frac{A}{t-2} + \frac{Bt+C}{t^2+t+1} =$$

$$= A(t^2+t+1) + (t-2)(Bt+C)$$

$$= t^2(A+B) + t(A+C-2B) + A-2C$$

$$\Rightarrow \begin{cases} t^2: (A+B) = 1 \\ t^1: A+C-2B = 0 \\ t^0: A-2C = -1 \end{cases} \Rightarrow \begin{cases} 3A+C = 2 \\ A-2C = -1 \\ A+B = 1 \end{cases} \Rightarrow \begin{cases} A = 3/7 \\ B = 4/7 \\ C = 5/7 \end{cases}$$

$$\text{⊖} \frac{1}{7} \int \left(\frac{3}{t-2} + \frac{4t+5}{t^2+t+1} \right) dt = \frac{1}{7} (3 \ln|t-2| + y_1)$$

$$y_1 = \int \frac{4t+5}{t^2+t+1} dt = 4 \int \frac{(t+\frac{1}{2}) + \frac{3}{4}}{(t+\frac{1}{2})^2 + \frac{3}{4}} dt = 2 \left(\int \frac{d((t+\frac{1}{2})^2 + \frac{3}{4})}{(t+\frac{1}{2})^2 + \frac{3}{4}} + \right.$$

$$\left. + \frac{3}{2} \int \frac{dt}{(t+\frac{1}{2})^2 + \frac{3}{4}} \right) = 2 \ln|(t+\frac{1}{2})^2 + \frac{3}{4}| + \frac{3}{2} \cdot \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{(t+\frac{1}{2})}{\sqrt{3}} + C$$

$$= 2 \ln|(t+\frac{1}{2})^2 + \frac{3}{4}| + \sqrt{3} \operatorname{arctg} \frac{2t+1}{\sqrt{3}} + C$$

Big nobis: $\frac{1}{7} (3 \ln|t-2| + 2 \ln|(t+\frac{1}{2})^2 + \frac{3}{4}| + \sqrt{3} \operatorname{arctg} \frac{2t+1}{\sqrt{3}}) + C$

Метод Остатков

№ 51 4

$$\int \frac{2t^2 + t - 6}{(t^2 - 2t + 5)^5 (t+3)^4} dt$$

$$\int \frac{P(x)}{Q(x)} dx = \int \frac{P_1(x)}{Q_1(x)} + \int \frac{P_2(x)}{Q_2(x)} dx$$

$$Q = Q_1 \cdot Q_2, \quad Q_1 = \text{HCD}(Q, Q')$$

$$Q_1 = (t^2 - 2t + 5)^4 \cdot (t+3)^3$$

$$\Rightarrow \int \frac{2t^2 + t - 6}{(t^2 - 2t + 5)^5 (t+3)^4} dt = \frac{At^{10} + Bt^9 + Ct^8 + Dt^7 + Et^6 + Ft^5 + Gt^4 + Ht^3 + It^2 + Jt + K}{(t^2 - 2t + 5)^4 (t+3)^3}$$

$$+ \int \frac{Pt^2 + Qt + R}{(t^2 - 2t + 5)(t+3)} dt \quad \text{14 вебогору}$$

$$\Rightarrow \frac{2t^2 + t - 6}{(t^2 - 2t + 5)^5 (t+3)^4} = \left(\frac{At^{10} + Bt^9 + Ct^8 + Dt^7 + Et^6 + Ft^5 + Gt^4 + Ht^3 + It^2 + Jt + K}{(t^2 - 2t + 5)^4 (t+3)^3} + \frac{Pt^2 + Qt + R}{(t^2 - 2t + 5)(t+3)} \right)'$$

$$\left(\frac{At^{10} + \dots + K}{(t^2 - 2t + 5)^4 (t+3)^3} + \frac{Pt^2 + Qt + R}{(t^2 - 2t + 5)(t+3)} \right)' = \frac{(At^{10} + \dots + K)' \cdot (t^2 - 2t + 5)^4 (t+3)^3 - (At^{10} + \dots + K) \cdot (4(t^2 - 2t + 5)^3 \cdot (2t - 2) \cdot (t+3)^3 + (t^2 - 2t + 5)^4 \cdot 3(t+3)^2)}{(t^2 - 2t + 5)^8 (t+3)^6} =$$

$$\frac{(At^{10} + \dots + K)' (t^2 - 2t + 5)(t+3) - (At^{10} + \dots + K) (4(2t - 2) \cdot (t+3) + (t^2 - 2t + 5) \cdot 3)}{(t^2 - 2t + 5)^5 (t+3)^4}$$

$$\Rightarrow 2t^2 + t - 6 = (10At^9 + 9Bt^8 + 8Ct^7 + 7Dt^6 + 6Et^5 + 5Ft^4 + 4Gt^3 + 3Ht^2 + 2It + J)(t^2 - 2t + 5)(t+3) - (At^{10} + Bt^9 + Ct^8 + Dt^7 + Et^6 + Ft^5 + Gt^4 + Ht^3 + It^2 + Jt + K) \cdot$$

$$(8(t-1)(t+3) + 3(t^2 - 2t + 5)) + (Pt^2 + Qt + R) \cdot (t^2 - 2t + 5)^4 (t+3)^3$$

$$2t^2 + t - 6 = (10At^9 + 9Bt^8 + 8Ct^7 + 7Dt^6 + 6Et^5 + 5Ft^4 + 4Gt^3 + 3Ht^2 + 2It + J)(t^3 + t^2 - t + 15) - (At^{10} + Bt^9 + Ct^8 + Dt^7 + Et^6 + Ft^5 + Gt^4 + Ht^3 + It^2 + Jt + K) \cdot (11t^2 + 10t + 9) + (Pt^2 + Qt + R)(t^8 - 8t^7 + 44t^6 - 152t^5 + 406t^4 - 760t^3 + 1100t^2 - 1000t + 625)(t^3 + 9t^2 + 27t + 27)$$

$$t^{13}: 0 = P$$

$$t^{12}: 0 = 10A + A - 8P + Q + 9P = 11A + P + Q$$

$$t^{11}: 0 = 10A + 9B + 2A - B + 27P - 72P + 44P + Q(9 - 8) + R = 12A + 8B - P + Q + R$$

$$t^2: 2 =$$

$$\begin{aligned}
 2t^2 + t - 6 &= (10At^9 + 9Bt^8 + 8Ct^7 + 7Dt^6 + 6Et^5 + 5Ft^4 + \\
 &+ 4Gt^3 + 3Ht^2 + 2It + J)(t^3 + t^2 - t + 15) - \\
 &- (At^{10} + Bt^9 + Ct^8 + Dt^7 + Et^6 + Ft^5 + Gt^4 + Ht^3 + It^2 + Jt + K) \cdot \\
 &\cdot (11t^2 + 10t - 9) + \\
 &+ (Pt^2 + Qt + R)(t^{11} + t^{10} - t^9 + 55t^8 + 10t^7 - 22t^6 + 1118t^5 - \\
 &- 658t^4 + 805t^3 + 8325t^2 - 10125t + 16875)
 \end{aligned}$$

$$\begin{cases}
 t^{13}: & 0 = P \\
 t^{12}: & 0 = -A + P + Q \\
 t^{11}: & 0 = -2B - P + Q + R \\
 t^{10}: & 0 = -A - B - 3C + 55P - Q + R \\
 t^9: & 0 = 150A - 2C - 4D + 10P + 55Q - R \\
 t^8: & 0 = 135B + C - 3D - 5E - 22P + 10Q + 55R \\
 t^7: & 0 = 120C + 2D - 4E - 6F + 1118P - 22Q + 10R \\
 t^6: & 0 = 105D + 3E - 5F - 7G - 658P + 1118Q - 22R \\
 t^5: & 0 = 90E + 4F - 6G - 8H + 805P - 658Q + 1118R \\
 t^4: & 0 = 75F + 5G - 7H - 9I + 8325P + 805Q - 658R \\
 t^3: & 0 = 60G + 6H - 8I - 10J - 10125P + 8325Q + 805R \\
 t^2: & 0 = 45H + 7I - 9J - 11K + 16875P - 10125Q + 8325R \\
 t^1: & 1 = 30I + 8J - 10K + 16875Q - 10125R \\
 t^0: & -6 = 15J + 9K + 16875R
 \end{cases}$$

$$\Rightarrow A = \dots, \quad P = 0, \quad Q = -\frac{201249}{8192000000}, \quad R = -\frac{170823}{16384000000}$$

$$y_1 = \int \frac{-\frac{201249}{8192000000}t - \frac{170823}{16384000000}}{(t^2 - 2t + 5)(t + 3)} dt =$$

$$= -\frac{1}{8192000000} \int \frac{201249t + 854115}{(t^2 - 2t + 5)(t + 3)} dt$$

y_2

$$MHK: \frac{201249t + 854115}{(t^2 - 2t + 5)(t+3)} = \frac{U}{t^2 - 2t + 5} + \frac{V}{t+3}$$

$$\text{Былобого: } \frac{933450 - 2029130t}{t^2 - 2t + 5} - \frac{1200(2501t + 3707)}{(t^2 - 2t + 5)^2} + \\ + \frac{6900(973t - 8205)}{(t^2 - 2t + 5)^3} + \frac{256000(317t - 705)}{(t^2 - 2t + 5)^4} + \\ + 62592 \ln|t^2 - 2t + 5| + \frac{16640}{t+3} - \frac{89600}{(t+3)^2} - \frac{76800}{(t+3)^3} - \\ - 125184 \ln|t+3| - 1256613 \arctg \frac{t-1}{2} \Big/ 8192000000 + C$$

Интервал для иррационального выражения

$$\boxed{NS3 \text{ г)}} \int \frac{dt}{t^2 \sqrt{t^2 + t - 1}} = \int \frac{dt}{t^3 \sqrt{1 + \frac{1}{t} - \frac{1}{t^2}}} = \left| \frac{1}{t} = x \right| \\ = - \int \frac{x dx}{\sqrt{1+x-x^2}} = - \int \frac{x dx}{\sqrt{-(x-\frac{1}{2})^2 + \frac{3}{4}}} = - \int \frac{x dx}{\sqrt{\frac{3}{4} - (x-\frac{1}{2})^2}} = \\ = - \left(\int \frac{(x-\frac{1}{2}) dx}{\sqrt{\frac{3}{4} - (x-\frac{1}{2})^2}} + \frac{1}{2} \int \frac{dx}{\sqrt{\frac{3}{4} - (x-\frac{1}{2})^2}} \right) = \\ = + \frac{1}{2} \sqrt{\frac{3}{4} - (x-\frac{1}{2})^2} - \frac{1}{2} \arcsin \frac{2(x-\frac{1}{2})}{\sqrt{3}} + C$$

$$\boxed{NS4 \text{ д)}} \int \frac{t \cdot \sqrt[3]{2+t}}{t + \sqrt[3]{2+t}} dt = \left| \begin{array}{l} \sqrt[3]{2+t} = x \\ 2+t = x^3 \\ t = x^3 - 2 \end{array} \right| \\ = \int \frac{(x^3 - 2) \cdot x \cdot 3x^2}{x^3 - 2 + x} dx = 3 \int \frac{x^6 - 2x^3}{x^3 + x - 2} dx = \\ = 3 \int \left(x^3 - x + \frac{x^2 - 2x}{(x-1)(x^2 + x + 2)} \right) dx = \frac{3}{4} x^4 - \frac{3}{2} x^2 + 3 \int \frac{x^2 - 2x}{(x-1)(x^2 + x + 2)} dx$$

$$MHK: \frac{3x^2 - 6x}{(x-1)(x^2 + x + 2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2 + x + 2} \Rightarrow y_1$$

$$A = -\frac{3}{4}, B = \frac{15}{4}, C = -\frac{3}{2}$$

$$\Rightarrow y_1 = -\frac{3}{4} \int \frac{dx}{x-1} + \frac{15}{4} \int \frac{x - \frac{3}{2}}{x^2 + x + 2} =$$

$$= -\frac{3}{4} \ln|x-1| + \frac{15}{8} \ln|x^2 + x + 2| - \frac{27}{4\sqrt{7}} \arctg \frac{2x+1}{\sqrt{7}} + C$$

$$\text{Былобого: } \frac{3}{4} x^4 - \frac{3}{2} x^2 - \frac{3}{4} \ln|x-1| + \frac{15}{8} \ln(x^2 + x + 2) - \\ - \frac{27}{4\sqrt{7}} \arctg \frac{2x+1}{\sqrt{7}} + C$$

Д/З NS0 г), д); NS1 б), NS3 е), NS4 а)