

Зачет 21.

Интеграл Ньютона - Лейбница.

- Звездочки 90 таблица интегралов
- Логические значения
- Рациональные значения
- Тригонометрические (гиперболические) функции
- Интегрирование по частям

$$\boxed{N40 a)} \int \frac{t - \sqrt{t} + 2\sqrt[3]{t}}{t} dt = \int (t^{-1/3} - t^{-1/2} + 2t^{-1/3}) dt =$$

$$= \int (t^{2/3} - t^{1/6} + 2t^{-1/2}) dt = t^{5/3} \cdot \frac{3}{5} - \frac{6}{7} t^{7/6} + 2 \cdot t^{1/2} \cdot \frac{12}{11} + C$$

$$\boxed{N40 б)} \int \left(\frac{1 + \sqrt{t}}{t} \right)^2 dt =$$

$$\boxed{N40 в)} \int \left(1 - \frac{1}{t^2} \right) \sqrt{t} \sqrt{t+1} dt = \int \left(1 - \frac{1}{t^2} \right) t^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} dt =$$

$$= \int \left(1 - \frac{1}{t^2} \right) t^{7/8} dt = \int (t^{7/8} - t^{-9/8}) dt = \frac{8}{15} t^{15/8} + 8 t^{-1/8} + C$$

$$\boxed{N40 г)} \int \frac{t^2}{1+t^2} dt = \int \left(1 - \frac{1}{1+t^2} \right) dt = t - \arctg t + C$$

$$\boxed{N40 д)} \int \frac{2-t^2}{t^2-1} dt =$$

$$\boxed{N40 е)} \int \frac{\sqrt{t^2+1} - \sqrt{t^2-1}}{\sqrt{t^2-1}} dt = \int \frac{dt}{\sqrt{t^2-1}} - \int \frac{dt}{\sqrt{t^2+1}} =$$

$$= \ln |t + \sqrt{t^2-1}| + C$$

$$y_1 = \frac{\operatorname{ch}^2 x - \operatorname{sh}^2 x}{\operatorname{sh}^2 x} = \frac{1}{\operatorname{sh}^2 x}, \quad t = \frac{e^x + e^{-x}}{2}$$

$$\textcircled{=} \int \frac{\operatorname{sh} x}{\operatorname{sh}^2 x} dx = \int \frac{1}{\operatorname{sh} x} dx = \ln |t + \sqrt{t^2-1}| + C \textcircled{=}$$

$$\textcircled{=} \left| t = \operatorname{ch} x = \frac{e^x + e^{-x}}{2} \Rightarrow e^{2x} - 2e^x t + 1 = 0 \right.$$

$$e^x = y \quad y^2 - 2ty + 1 = 0 \quad D = 4t^2 - 4$$

$$y_{1,2} = \frac{2t \pm 2\sqrt{t^2-1}}{2} = t \pm \sqrt{t^2-1}$$

$$x = \ln y = \ln(t \pm \sqrt{t^2-1}) = \pm \ln(t + \sqrt{t^2-1})$$

$$\textcircled{=} \ln(t + \sqrt{t^2-1})$$

$$\text{Ответ: } \ln(t + \sqrt{t^2-1}) - \ln(t + \sqrt{t^2+1}) + C$$

2 cascad odz. J₁

zamena: $t = \frac{1}{\cos x}$, $dt = \frac{\sin x}{\cos^2 x} dx$,

$$\sqrt{t^2 - 1} = \sqrt{\frac{1}{\cos^2 x} - 1} = \sqrt{\frac{\sin^2 x}{\cos^2 x}} = \tan x$$

$$J_1 = \int \frac{\sin x}{\cos^2 x \cdot \tan x} dx = \int \frac{dx}{\cos x} = \int \frac{d(x + \frac{\pi}{2})}{\sin(x + \frac{\pi}{2})} =$$

$$= \int y = x + \frac{\pi}{2} \int \frac{dy}{\sin y} = \int \frac{dy}{2 \sin \frac{y}{2} \cos \frac{y}{2}} = \frac{1}{2} \int \frac{dy}{\tan \frac{y}{2} \cos^2 \frac{y}{2}} =$$

$$= \int \frac{d(\tan \frac{y}{2})}{\tan \frac{y}{2}} = \ln |\tan \frac{y}{2}| + C = \ln |\tan \frac{x + \frac{\pi}{2}}{2}| + C$$

$$= \ln \left| \frac{\tan \frac{x}{2} + 1}{1 - \tan \frac{x}{2}} \right| + C = \left| \tan \frac{x}{2} = \frac{\sin x}{1 + \cos x} \right| =$$

$$= \ln \left| \frac{\sin x + 1 + \cos x}{1 + \cos x - \sin x} \right| + C = \ln \left| \frac{\tan x + \frac{1}{\cos x} + 1}{-\tan x + \frac{1}{\cos x} + 1} \right| + C$$

$$= \ln \left| \frac{1 + t + \sqrt{t^2 - 1}}{1 + t - \sqrt{t^2 - 1}} \right| + C =$$

$$= \ln \left| \frac{(1 + t + \sqrt{t^2 - 1})^2}{(1 + t)^2 - (t^2 - 1)} \right| + C =$$

$$= \ln \left| \frac{2t^2 + 2t + 2(1 + t)\sqrt{t^2 - 1}}{2 + 2t} \right| + C = \ln |t + \sqrt{t^2 - 1}| + C$$

N40 m $\int \frac{2^{t+1} - 5^t - 10^{t-1}}{10^t} dt =$

N40 z $\int \frac{t}{t^4 + 2t^2} dt = \left| t^2 = u \right| = \frac{1}{2} \int \frac{du}{u^2 + 2u} =$

$$= \frac{1}{2} \int \frac{du}{(u+1)^2 - 1} = \frac{1}{4} \ln \left| \frac{u+1-1}{u+1+1} \right| + C$$

$$= \frac{1}{4} \ln \left| \frac{u}{u+2} \right| + C = \frac{1}{4} \ln \left| \frac{t^2}{t^2 + 2} \right| + C$$

N40 x) $\int (\sqrt[3]{t} - \sqrt[4]{t})^{12} dt = \int t^{11/12} - t^{10/12} dt = \int t^{11/12} - t^{5/6} dt = \frac{12}{11} t^{21/12} - \frac{12}{5} t^{11/6} + C$

$= \int (y^4 - y^3)^{12} \cdot 12y'' dy = 12 \int y^{47} (y-1)^{12} dy =$

$= 12 \int y^{47} (C_{12}^0 y^{12} - C_{12}^1 y^{11} + C_{12}^2 y^{10} - C_{12}^3 y^9 + C_{12}^4 y^8 - C_{12}^5 y^7 + C_{12}^6 y^6 - C_{12}^7 y^5 + C_{12}^8 y^4 - C_{12}^9 y^3 + C_{12}^{10} y^2 - C_{12}^{11} y + C_{12}^{12}) dy$

$= 12 \int (C_{12}^0 y^{59} - C_{12}^1 y^{58} + C_{12}^2 y^{57} - C_{12}^3 y^{56} + C_{12}^4 y^{55} - C_{12}^5 y^{54} + C_{12}^6 y^{53} - C_{12}^7 y^{52} + C_{12}^8 y^{51} - C_{12}^9 y^{50} + C_{12}^{10} y^{49} - C_{12}^{11} y^{48} + C_{12}^{12} y^{47}) dy$

$= 12 \int \left(\sum_{k=0}^{12} C_{12}^k y^{47+12-k} \right) dy = 12 \sum_{k=0}^{12} C_{12}^k \frac{y^{47+12-k+1}}{47+12-k+1} + C$

$= 12 \sum_{k=0}^{12} C_{12}^k \frac{y^{60-k}}{60-k} + C = 12 \sum_{k=0}^{12} C_{12}^k \frac{t^{\frac{60-k}{12}}}{60-k} + C$

N40 y) $\int \frac{3t^3 - 5t^2 + t + 2}{t+1} dt =$

~~$$\begin{array}{r} 3t^3 - 5t^2 + t + 2 : t+1 \\ 3t^3 + 3t^2 \\ \hline -8t^2 + t + 2 \\ -8t^2 - 8t \\ \hline 9t + 2 \\ 9t + 9 \\ \hline -7 \end{array}$$~~

$$\begin{array}{r} 3t^3 - 5t^2 + t + 2 : t+1 \\ 3t^3 + 3t^2 \\ \hline -8t^2 + t + 2 \\ -8t^2 - 8t \\ \hline 9t + 2 \\ 9t + 9 \\ \hline -7 \end{array}$$

$$= \int \left(3t^2 - 8t + 7 - \frac{7}{t+1} \right) dt = t^3 - 4t^2 + 7t - 7 \ln|t+1| + C$$

N40 u) $\int \sinh t \cosh t dt = \int \cosh t d(\sinh t) = \frac{\cosh^2 t}{2} + C$

N40 v) $\int \cosh^2 t dt = \int \left(\frac{e^t + e^{-t}}{2} \right)^2 dt = \int \frac{e^{2t} + 2 + e^{-2t}}{4} dt = \int \left(\frac{1}{2} \cosh 2t + \frac{1}{2} \right) dt = \frac{1}{2} \left(\frac{\sinh 2t}{2} + t \right) + C$

N41 g) Булоп. лінкавы гарма, зварна інерца.

$$\int \sqrt[15]{1+4t} dt = \left| \begin{array}{l} 1+4t = y \\ t = \frac{y-1}{4}, dt = \frac{dy}{4} \end{array} \right| =$$

$$= \int y^{1/15} \cdot \frac{dy}{4} = \frac{1}{4} y^{16/15} \cdot \frac{15}{16} + C = \frac{15}{64} (1+4t)^{16/15} + C$$

N41 e) $\int (t+2)\sqrt{t-2} dt = \left| \begin{array}{l} t-2 = y \\ dt = dy \end{array} \right| =$

$$= \int (y+4)\sqrt{y} dy = \int (y^{3/2} + 4y^{1/2}) dy = y^{5/2} \cdot \frac{2}{5} + 4y^{3/2} \cdot \frac{2}{3} + C$$

$$= (t-2)^{5/2} \cdot \frac{2}{5} + \frac{8}{3} (t-2)^{3/2} + C$$

N41 з) $\int (2t-1)(t-1)^7 dt =$

N41 к) $\int \frac{dt}{a^2+b^2t^2} = \frac{1}{a^2} \int \frac{dt}{1+(\frac{bt}{a})^2} =$

$$= \left| \begin{array}{l} \frac{bt}{a} = y \\ dy = \frac{b}{a} dt \end{array} \right| = \frac{1}{a^2} \cdot \frac{a}{b} \int \frac{dy}{1+y^2} = \frac{1}{ab} \operatorname{arctg} y + C$$

$$= \frac{1}{ab} \operatorname{arctg} \frac{bt}{a} + C$$

N42 a) Булоп. паўкошыны гарма, одз. інерца.

$$\int t^4 \sqrt{1+t^5} dt = \left| \begin{array}{l} 1+t^5 = y \\ 5t^4 dt = dy \end{array} \right| =$$

$$= \frac{1}{5} \int \sqrt{y} dy = \frac{1}{10\sqrt{y}} + C = \frac{1}{10\sqrt{1+t^5}} + C$$

N42 б) $\int \frac{t^3}{1+t^4} dt =$

N42 2) $\int \frac{t}{a^2+b^2t^2} dt =$

N42 7)

$$\int \frac{t^7}{\sqrt{1-t^{16}}} dt =$$

N42 x)

$$\int \frac{t^3}{t^4 - t^2 + 2} dt = \left/ \begin{array}{l} t^2 = y \\ 2t dt = dy \end{array} \right/$$

$$= \frac{1}{2} \int \frac{y}{y^2 - y + 2} dy = \frac{1}{2} \left(\int \frac{y - \frac{1}{2}}{y^2 - y + 2} dy + \int \frac{\frac{1}{2}}{y^2 - y + 2} dy \right)$$

$$= \cancel{\frac{1}{2} \int} = \frac{1}{4} \int \frac{d(y^2 - y + 2)}{y^2 - y + 2} + \frac{1}{4} \int \frac{dy}{y^2 - y + 2} =$$

$$= \frac{1}{4} \ln |y^2 - y + 2| + \frac{1}{4} J_2 \quad \textcircled{=}$$

$$J_2 = \int \frac{dy}{(y - \frac{1}{2})^2 + \frac{7}{4}} = \frac{1}{\sqrt{7}} \int \frac{2 dy}{(y - \frac{1}{2})^2 + \frac{7}{4}} = \frac{2}{\sqrt{7}} \operatorname{arctg} \frac{(y - \frac{1}{2}) \cdot 2}{\sqrt{7}} + C$$

$$= \frac{2}{\sqrt{7}} \operatorname{arctg} \left(\frac{2y - 1}{\sqrt{7}} \right) + C$$

$$\textcircled{=} \frac{1}{4} \ln |t^4 - t^2 + 2| + \frac{1}{4} \cdot \frac{2}{\sqrt{7}} \operatorname{arctg} \left(\frac{2t^2 - 1}{\sqrt{7}} \right) + C$$

N43 H)

Вукоп. Тригоном / тригд. нис еракобен,
одн. интеграл

$$\int \frac{\sin t \cos t}{\sin^4 t + \cos^4 t} dt = \int \frac{\sin t \cos t}{(\sin^2 t + \cos^2 t)^2 - 2\sin^2 t \cos^2 t} dt =$$

$$= \int \frac{\sin t \cos t dt}{1 - 2\sin^2 t \cdot \cos^2 t} = \frac{1}{2} \int \frac{\sin 2t}{1 - \frac{1}{2} \sin^2 2t} dt =$$

$$= \frac{1}{2} \int \frac{\sin 2t}{1 - \frac{1}{2}(1 - \cos^2 2t)} dt = -\frac{1}{4} \int \frac{d(\cos 2t)}{\frac{1}{2} + \frac{1}{2} \cos^2 2t} =$$

$$= \cancel{\frac{1}{2}} \left/ \begin{array}{l} y = \cos 2t \\ dy = d(\cos 2t) \end{array} \right/ = -\frac{1}{2} \int \frac{dy}{1 + y^2} =$$

$$= -\frac{1}{2} \operatorname{arctg} y + C = -\frac{1}{2} \operatorname{arctg} \cos 2t + C$$

N 44 а) Обяснителен израз, външност.
ръкописна подготовка.

Объяснить интеграл, воспольст.
разног. типы подсказок

$$\int \frac{\ln^2 t}{t} dt =$$

N 44 2

$$\int \frac{dt}{\sqrt{1-t^2} \cdot \arcsin t} =$$

N 44 n

$$\int \frac{e^t}{1+e^t} dt =$$

N 44 n

$$\int \frac{t \ln t}{\sqrt{1+t^2}} dt =$$

9/3

N40 K) H) P) Y) ;

N41 d) m) p) x) ;

N42 3) 90) 0)

143 k) o) n) p) ;