

Зачет № 11. Границы функций.

N 135 4) Значения гранич: $\lim_{x \rightarrow 0} \frac{(1+mx)^n - (1+nx)^m}{x^2} = \frac{0}{0} / \text{невозм. } \frac{0}{0} / \text{L'Hopital}$

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2}x^2 + o(x^2)$$

$$\lim_{x \rightarrow 0} \frac{1 + nm x + \frac{n(n-1)}{2}(mx)^2 + o(x^2) - 1 - nm x - \frac{m(m-1)}{2}(nx)^2}{x^2} = \frac{n(n-1)m^2 - m(m-1)n^2}{2} = \frac{nm(n-m)}{2}$$

$$\frac{n(n-1)m^2 - m(m-1)n^2}{2} = \frac{-nm^2 + mn^2}{2} = \frac{mn(n-m)}{2}$$

N 135 2) $\lim_{x \rightarrow 8} \frac{\sqrt{9+2x} - 5}{\sqrt[3]{x} - 2 + (x-8)^3} = \frac{0}{0} / \text{L'Hopital}$

$$\begin{aligned} &= \lim_{t \rightarrow 0} \frac{\sqrt{9+2(8+t)} - 5}{\sqrt[3]{8+t} - 2 + t^3} = \lim_{t \rightarrow 0} \frac{5(1 + \frac{2t}{25})^{1/2} - 5}{2(1 + \frac{t}{8})^{1/3} - 2 + t^3} \\ &= \lim_{t \rightarrow 0} \frac{5(1 + \frac{t}{25} + o(t)) - 5}{2(1 + \frac{t}{24} + o(t)) - 2 + t^3} = \lim_{t \rightarrow 0} \frac{\frac{t}{5}}{\frac{t}{12}} = \frac{12}{5} \end{aligned}$$

N 135 2) $\lim_{x \rightarrow 0} \frac{(\sqrt{1+x^2} + x)^n - (\sqrt{1+x^2} - x)^n}{x} = \frac{0}{0} / \text{L'Hopital}$

$$\lim_{x \rightarrow 0} \frac{(1 + \frac{x^2}{2} + o(x^2) + x)^n - (1 + \frac{x^2}{2} + o(x^2) - x)^n}{x}$$

$$\lim_{x \rightarrow 0} \frac{(1 + x + o(x))^n - (1 - x + o(x))^n}{x}$$

$$\lim_{x \rightarrow 0} \frac{1 + nx - (1 - nx)}{x} = \lim_{x \rightarrow 0} \frac{2nx}{x} = 2n$$

N 136 4) $\lim_{x \rightarrow \pi} \frac{\sin 3x}{\sin 2x} = \frac{0}{0} / \text{L'Hopital} = \lim_{t \rightarrow 0} \frac{\sin(3t + 3\pi)}{\sin(2t + 2\pi)}$

$$\lim_{t \rightarrow 0} \frac{-\sin 3t}{\sin 2t} = \lim_{t \rightarrow 0} \frac{-3t + o(t)}{2t + o(t)} = -\frac{3}{2}$$

$$\boxed{N135 \text{ u)}} \quad \lim_{x \rightarrow +\infty} \frac{\sqrt{x + \sqrt{x + \sqrt{x}}}}{\sqrt{x+1}} = \left| \frac{\infty}{\infty} \right| =$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \sqrt{\frac{x + \sqrt{x}}{x^2}}}}{\sqrt{1 + \frac{1}{x}}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x^3}}}}}{\sqrt{1 + \frac{1}{x}}} = 1$$

$$\boxed{N136 \text{ u)}} \quad \lim_{x \rightarrow 0} \frac{\sin(x \sin(2x \cdot \sin 3x))}{x^3} = \left| \frac{0}{0} \right| =$$

$$= \lim_{x \rightarrow 0} \frac{\sin(x \sin(2x \cdot 3x))}{x^3} = \lim_{x \rightarrow 0} \frac{\sin(x \cdot 6x^2)}{x^3} =$$

$$= \lim_{x \rightarrow 0} \frac{6x^3}{x^3} = 6$$

$$\boxed{N137 \text{ d)}} \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3} = \left| \frac{0}{0} \right| =$$

$$\stackrel{E}{=} \lim_{x \rightarrow 0} \frac{1 + \tan x - 1 - \sin x}{x^3 (\sqrt{1 + \tan x} + \sqrt{1 + \sin x})} = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3 (\sqrt{1 + \tan x} + \sqrt{1 + \sin x})} =$$

$$\stackrel{E}{=} \lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot \left(\frac{1 - \cos x}{x^2} \right) \cdot \frac{1}{\sqrt{1 + \tan x} + \sqrt{1 + \sin x}} = \frac{1}{4}$$

$\searrow 1$ $\searrow \frac{1}{2}$ $\searrow \frac{1}{2}$

$$\boxed{N137 \text{ g)}} \quad \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} - \cos x + (1+x)^{5/4} - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x + o(x^2)}{x} - (1 + o(x)) + 1 + \frac{5}{4}x + o(x) - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{1 + o(x) - 1 + o(x) + 1 + \frac{5}{4}x + o(x) - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{5}{4}x + o(x)}{x} = \frac{5}{4}$$

$$\boxed{N138 \text{ u)}} \quad \lim_{x \rightarrow a} \frac{a^x - a^a}{x^a - a^a} = \left| \frac{0}{0} \right| = \lim_{x \rightarrow a} \frac{e^{x \ln a} - e^{a \ln a}}{e^{a \ln x} - e^{a \ln a}}$$

$$\stackrel{L}{=} \lim_{t \rightarrow 0} \frac{a^{a+t} - a^{a+t}}{(a+t)^a - a^{a+t}} =$$

$$= \lim_{t \rightarrow 0} \frac{e^{(a+t) \ln a} \cdot \ln a - e^{(a+t) \ln a}}{e^{(a+t) \ln a} - a^{a+t}} =$$

$$= \lim_{t \rightarrow 0} \frac{a^a ((1 + \frac{t}{a})^a - e^{t \ln a})}{a^a (e^{(a+t) \ln a} \ln a - a^{a+t})} =$$

$$= \lim_{t \rightarrow 0} \frac{a^a (e^{(a+t) \ln a} \ln a - a^{a+t})}{a^a (1 + t + o(t) - 1 - t \ln a)} =$$

$$\begin{aligned}
 & \textcircled{=} \lim_{t \rightarrow 0} \frac{a^a - a \left(e^{a \ln a (1+t)} - e^{a \ln a} \right) \ln a (1+t)^a - a^a}{-e} \\
 & = \lim_{t \rightarrow 0} \frac{a^a - a \left(e^{a \ln a} e^{a \ln a t} (e^{a \ln a} - 1) - e^{a \ln a} ((1+t/a)^a - 1) \right)}{t(1-\ln a)} \\
 & = \lim_{t \rightarrow 0} \frac{a^a - a \left(e^{a \ln a} \cdot e^{a \ln a} \cdot t \ln a - e^{a \ln a} \cdot \ln a \cdot t \right)}{t(1-\ln a)} \\
 & = \lim_{t \rightarrow 0} \frac{a^a - a \left(1 + \ln a \cdot e^{a \ln a} \cdot \ln a \cdot t - 1 - a^a \cdot \ln a \cdot t \right)}{t(1-\ln a)} \\
 & = \lim_{t \rightarrow 0} \frac{a^a - a \left((\ln a)^2 a^a - a^a \cdot \ln a \right)}{1 - \ln a} = \frac{a^a \cdot \ln a \cdot (\ln a - 1)}{1 - \ln a} \\
 & = -a^a \cdot \ln a
 \end{aligned}$$

$$\begin{aligned}
 & \boxed{N138n} \quad \lim_{x \rightarrow 0} \frac{\text{sh}(x^2+x) - \text{sh}(x^2-x)}{1 - \text{ch}x} = \frac{0}{0} \\
 & = \lim_{x \rightarrow 0} \frac{x + x^2 + o(x^2) - (-x + x^2 + o(x^2))}{-x^2/2 + o(x^2)} = \lim_{x \rightarrow 0} \frac{2x}{-x^2/2} = \infty
 \end{aligned}$$

$$\begin{aligned}
 & \boxed{N139H} \quad \lim_{x \rightarrow +\infty} \frac{\text{sh}\sqrt{x^2+x} - \text{sh}\sqrt{x^2-x}}{\text{ch}x} = \frac{\infty - \infty}{\infty} \\
 & = \lim_{x \rightarrow +\infty} \frac{e^{\sqrt{x^2+x}} - e^{-\sqrt{x^2-x}}}{e^x + e^{-x}} \\
 & = \lim_{x \rightarrow +\infty} \frac{e^{\sqrt{x^2+x}}}{e^x} = \lim_{x \rightarrow +\infty} e^{\sqrt{x^2+x} - x} \\
 & = \lim_{x \rightarrow +\infty} e^{x(\sqrt{1+1/x} - 1)} = \lim_{x \rightarrow +\infty} e^{\frac{\sqrt{1+t} - 1}{t}} \\
 & = \lim_{t \rightarrow 0} e^{\frac{1+t-1}{2t}} = \lim_{t \rightarrow 0} e^{\frac{1-t-1}{t}} = \lim_{t \rightarrow 0} e^{-\frac{1}{t}} = 0
 \end{aligned}$$

N 140 K $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$ 1^∞ - неопределенность. $\frac{v(x)}{u(x)} = 0$ $\frac{v(x) \ln u(x)}{u(x)}$

⑤ $\lim_{x \rightarrow 0} e^{\frac{\ln \cos x}{x^2}} = \lim_{x \rightarrow 0} e^{\frac{\ln(1 - \frac{x^2}{2} + o(x^2))}{x^2}} =$

$= \lim_{x \rightarrow 0} e^{-\frac{\frac{x^2}{2} + o(x^2)}{x^2}} = e^{-1/2} = \frac{1}{\sqrt{e}}$

N 143 a) 1) Знаем односторонние пределы в т. x_0

$f(x) = [x], x_0 = 1$

Решение. $\lim_{x \rightarrow x_0 - 0} f(x) = \lim_{x \rightarrow 1 - 0} [x] = 0$

$\lim_{x \rightarrow x_0 + 0} f(x) = \lim_{x \rightarrow 1 + 0} [x] = 1$

$\Rightarrow \nexists \lim_{x \rightarrow 1} [x]$

2) $x_0 = 1,0001 \quad \exists \lim_{x \rightarrow 1,0001} [x] = 1$

3) $x_0 = \sqrt{2} \quad \exists \lim_{x \rightarrow \sqrt{2}} [x] = 1$

N 145 b) Доказать существование предела

$\lim_{x \rightarrow -0} \frac{2^x}{4 - 3^{-1/x}}$

N 149 a) Выяснить порядок бесконечности при

$x \rightarrow -\infty$ f при $x \rightarrow 0$, если

$f(x) = \ln(1+x^2) - 2\sqrt[3]{e^x - 1}$

Решение $f(x) = x^2 + o(x^2) - 2x^{2/3} = -2x^{2/3} + o(x^{2/3})$

$\Rightarrow n = 2/3$

2/3 N 135 4) b), N 136 a), N 137 a), N 138 m) c)

N 139 K), N 140 B), N 143 A), N 145 4)

N 145 02)